SHEAR STRENGTH PREDICTION OF REINFORCED CONCRETE SHALLOW BEAMS WITHOUT SHEAR REINFORCEMENTS

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Abstract: This paper presents two design equations to predict the shear strength of reinforced concrete shallow beams without shear reinforcements. The proposed equations were derived from two techniques: nonlinear regression analysis and artificial neural network analysis. The analysis were carried out using 279 test results of shallow beams available in the literature with wide range of geometrical and material properties. The proposed equations consider the influence of concrete compressive strength, flexural reinforcement, shear span-effective depth ratio, and the beam’s size. The calculations of the proposed equations are compared with those of the current codes of practice and those available in the literature, and they result as the best fitting to the available tests results.

Keywords: Reinforced Concrete, Shear Strength, Shallow Beams, Regression Analysis, Artificial Neural Network, Design.

The prediction of the bearing capacity of reinforced concrete shallow beams is a critical aspect in the design of structures. This paper presents two design equations to predict the shear strength of reinforced concrete shallow beams without shear reinforcements. The proposed equations were derived from two techniques: nonlinear regression analysis and artificial neural network analysis. The analysis were carried out using 279 test results of shallow beams available in the literature with wide range of geometrical and material properties. The proposed equations consider the influence of concrete compressive strength, flexural reinforcement, shear span-effective depth ratio, and the beam’s size. The calculations of the proposed equations are compared with those of the current codes of practice and those available in the literature, and they result as the best fitting to the available tests results.

Keywords: Reinforced Concrete, Shear Strength, Shallow Beams, Regression Analysis, Artificial Neural Network, Design.

1. Introduction

Reinforced concrete shallow beams can be defined as beams with a shear span to effective depth ratio equal to, or greater than 2.5 [1]. Despite of being widely used in constructions, their shear strength equations available in current code of practice: the American code [2], ACI318-14, the British Standard [3], BS8110-97, and the European code [4], EC2, are empirical in nature, and they were derived from statistical analysis to fit the test results available at the time of derivation.
Furthermore, the equations in these codes consider various parameters with their limitations, causing inconsistency in the shear capacity predictions [5-7].

The objective of this study is to use a large number of previous test results of shallow beams without shear reinforcements to propose reliable equations that agree favorably with the test results. Two approaches are employed to fit the test results: the nonlinear regression and the artificial neural network. From which, to derive two simple equations to predict the shear strength of shallow beams without shear reinforcements. Both equations account for the concrete compressive strength, longitudinal reinforcement, the effective depth of a beam, and the shear span-depth ratio. To examine the performance of the proposed equations, their predictions are compared with the existing equations for shear strength of shallow beams with no shear reinforcements.

2. Application of Artificial Neural Networks (ANNs)

The artificial neural networks (ANN) is a powerful mathematical tool that operates in a manner similar to that of biological neurons system. Scholars have implemented the artificial neural networks (ANNs) as an alternative to conventional analytical techniques which are frequently constrained by firm assumptions of normality, linearity, homogeneity, etc. Several neural networks models were found in the literature built to investigate the shear strength of reinforced concrete beams shallow beams without shear reinforcements [7-17]. Elsanadedy et al. [7], Oreta [8], Cladera and Mari [9], El-Chabib et al. [10], Seleemah [11], Jung and Kim [12], and Keskin [17] applied the artificial neural networks (ANNs) to predict the shear behavior of reinforced concrete shallow beams without shear reinforcements. While, Cladera and Mari [13], El-Chabib et al. [14], Abdalla et al. [15], and Mansour et al. [16] applied the artificial neural networks (ANNs) to predict the shear behavior of reinforced concrete shallow beams with shear reinforcements.

It is important to note that all of these studies were conducted to investigate the shear behavior of shallow beams by carrying out a parametric study to examine the effect of the geometrical and material properties on the shear strength.

Unlike the aforementioned studies, the present neural network model is built to determine the functional relationship between test parameters (input parameters). That is, the weight of each input is calculated using the connecting weight algorithm [18] to determine its contribution with respect to the other inputs and the corresponding shear strength. From which, to derive a simple design equation to predict the shear strength of shallow beams without shear reinforcements.

3. Existing Shear Equations For Shallow Beams

Table 1 summarize the selected design expressions provided by the current codes of practice for the shear strength of shallow beams with the material safety factor being set to unity. In addition to codes’ equations, selected design expressions from literature are also included in this table. A critical review about the design expressions provided by the current codes indicates that each expression has adopted significantly different parameters along with their limitations. To be precise, ACI318-14 [2] shear equation
considers the influence of concrete strength only. Equations provided by BS8110 [3] and EC2 [4], however, consider the influence of concrete strength, the flexural reinforcement, and the beam’s depth. While, all of these codes ignored the influence of shear span to effective depth ratio. Such differences in considering the influencing parameters would probably cause inconsistency in the shear strength predictions. Furthermore, the imposed limitations on the influencing parameters indicate that they were derived from limited range of test results.

Similar to codes’ equations, the existing equations in the literature are also varied in terms of influencing parameters. The equation of Niwa et al. [19] consider the influence of concrete strength, the flexural reinforcement, the beam’s effective depth, and the shear span-effective depth ratio. While, the equation of Collins and Kuchma [20] ignores the influence of flexural reinforcement, Rebeiz’s equation [21] ignores the size effect on shear strength, and that Tureyen and Frosch [22] ignores the effect of size and the shear span-effective depth ratio.

### Table 1. Selected shear strength equations

<table>
<thead>
<tr>
<th>Reference</th>
<th>Shear strength equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI318-14[2]</td>
<td>( v_c = 0.17\lambda \sqrt{\bar{f}_c} ) ( \lambda = 1 ) for normal concrete, ( \lambda = 0.75 ) &amp; 0.85 for lightweight concrete</td>
</tr>
<tr>
<td>BS8110[3]</td>
<td>( v_c = 0.79\frac{2}{\rho f_{cu}} \sqrt[4]{\frac{400}{d}} ) ( 4 \frac{400}{d^4} \geq 0.67, \rho \leq 3%, f_{cu} \leq 40 \text{ MPa} )</td>
</tr>
<tr>
<td>EC2[4]</td>
<td>( v_c = 0.18 k \sqrt[3]{\rho f_{c'}} ) ( k_d = 1 + \frac{200}{d} \leq 2, \rho \leq 0.02 )</td>
</tr>
<tr>
<td>Niwa et al.[19]</td>
<td>( v_c = 0.2 \sqrt[3]{\frac{1000}{d}} \left( 0.75 + \frac{1.4}{k_d} \right) )</td>
</tr>
<tr>
<td>Collins and Kuchma[20]</td>
<td>( v_c = \frac{0.245}{1275 + 5E} \sqrt{\bar{f}_c} ) ( S_E = \frac{35S_x}{(a + 16)} \cdot S_x = 0.9 \text{ d} )</td>
</tr>
<tr>
<td>Rebeiz [21]</td>
<td>( v_c = 0.4 + \frac{d}{\bar{f}_c \rho \sqrt{2.7 - 0.4A_D}} ) ( A_D = 2.5 \text{ for } \frac{d}{\bar{f}_c} \geq 2.5 )</td>
</tr>
<tr>
<td>Tureyen and Frosch [22]</td>
<td>( v_c = \frac{5}{12} k \sqrt{\bar{f}<em>c} ) ( k = \sqrt{(n\rho)^2 + (2n\rho) - n\rho} ) ( n = E</em>{st}/E_c, E_{st} = 200000 \text{ MPa} ) ( E_c = 4700\sqrt{\bar{f}_c} ) [2]</td>
</tr>
</tbody>
</table>

### 4. Shear Strength Database

A total of 279 test results of reinforced concrete shallow beams with no shear reinforcements were collected from previous experimental investigations [20, 23-56]. All beams were simply supported and subjected to point loadings. The test results were initially employed in the nonlinear regression analysis to derive the first shear equation. Then, they were employed to train, test, and validate the neural network model to derive the second shear equation.

The collected beams cover wide range of test variables. The concrete compressive strength ranged from 14 MPa to 110.9 MPa, the flexural reinforcement ratio ranged from 0.25% to 6.62%, the effective depth of beams ranged from 110 mm to 1200 mm, and the shear span to effective depth ratio ranged from 2.5 to 8.04. Table 2 provides detailed information about the beams used in this study.
5. Building the Neural Network Model

The present neural network (NN) model is built to predict the shear strength of shallow beams without shear reinforcements using MATLAB R2013b [57]. In general, the architecture of a NN model composed of input layer, one or more hidden layer(s), and output layers. The input layer composed of a number of neurons (usually represents the independent test parameters) plus one neuron have a value of one termed bias [58].
A trial and error approach is used to find the suitable number of hidden layers and their neurons that give best predictions. It is important to note that increasing the number of hidden layers or increasing the number of neurons in these layers increases the training time, prevent the model to generalized, and do not improve the predictions [58].

This model adopts the feed-forward, back propagation algorithm with one input layer, two hidden layers, and one output layer, see Fig 1. The input layer composed of four neurons and the hidden layers composed of fourteen and five neurons, respectively. The model adopted the tan-sigmoid transform function in hidden layers and the linear transform function in the output layer. Early stopping technique is also adopted to maintain generalization of the NN model and to avoid over-fitting [58]. The test data is divided into three subdivisions: training, validation, and testing. The training subdivision composed of 70% of the test data, and the remainder are equally divided between validation and testing subdivisions.

All variables influencing the shear strength are considered [7-17]. The test variables include the concrete compressive strength, the flexural reinforcement ratio, the effective depth of a beam, and the shear span-depth ratio. The NN model agreed very well the test results. The correlation coefficient (R) between predicted, $v_{\text{pred}}$, and test results, $v_{\text{test}}$, was 0.975, as shown in Fig 2.
6. Proposed Equations

Two design equations are proposed to predict the shear strength of shallow beams without shear reinforcement using nonlinear regression analysis and neural network analysis. The first equation is derived from the nonlinear regression analysis conducted using SPSS Statistics 22 [59]. The shear strength is made a function of concrete compressive strength, flexural reinforcement ratio, effective depth of a beam, and the shear span-depth ratio. The contribution of these parameters is calibrated with the test results of 279 concrete shallow beams from literature. This equation is expressed as follows:

\[
v_{pred} = 1.33 \frac{\sqrt[3]{f_c \rho}}{\sqrt{d}}
\]  
Eq. (1)

The second equation is derived from the neural network analysis. It also consider the same test variables in equation (1). The contribution of each parameter with regards to other is determined using the connecting weights algorithm conducted after the achieving the successful NN model. This equation can be expressed as follows:

\[
v_{pred} = 2.09 f_c^{0.24} \rho^{0.35} \left( \frac{\alpha}{d} \right)^{-0.18} d^{-0.23}
\]  
Eq. (2)

For both equations, the predicted shear strength, \(v_{pred}\), and concrete compressive strength, \(f_c\) are in MPa, the shear span, \(a\), and the effective depth of a beam, \(d\), are in mm, and the flexural reinforcement ratio, \(\rho\), in percentage.
7. Validation of The Proposed Equations

Both equations were applied to predict the test shear strength of 279 concrete beams from literature. Both equations achieved good agreements with the test results, as shown in Fig 3. For equation (1), the mean of predicted strength to test was 1.00 with a standard deviation of 0.14. For equation (2), the mean of predicted strength to test was 1.00 with a standard deviation of 0.13.

8. Comparison Between The Proposed and Existing Equations

The proposed equations are applied to predict the shear strength of 279 reinforced concrete shallow beams from literature along with the seven existing equations listed in Table 1. The results are presented separately for each equation in Fig 3 in terms of predicted strength to test. It can be inferred that the existing equations provide inconsistent predictions, scattered on each side of the equality line. In general, the ACI318-14 [2], BS8110 [3], EC2 [4], Niwa et al. [19], Collins and Kuchma [20], and Tureyen and Frosch [22] equations were conservative; and the equation of Rebiz [21] was unsafe. Unlike the existing equations, the proposed ones agreed well with the test results. This would be expected since the proposed equations consider all governing variables that influence the shear strength of shallow beams and they were calibrated with large number of test beams covering wide range of test variables.

To examine the effect of the governing variables on the predictions of the existing and proposed equations, each of governing variables is plotted against the ratio of predicted strength to test, see Figs 4 to 7.
8.1. Effect of Concrete Compressive Strength

Fig 4 shows the effect of concrete compressive strength on the strength predictions of the existing and the proposed equations. The majority of the test beams (76%) were cast from normal concrete with compressive strength below 45 MPa (210 of 277). This indicates that the equations of ACI318-14 [2] and Collins and Kuchma [20] were scattered with the concrete strength. The BS8110 [3] and EC2 [4] equations provided consistent predictions with the strength predictions for concrete strength below 40 MPa.
and scattered predictions for concrete strength above than 40 MPa. The equations of Niwa et al. [19] and Tureyen and Frosch [22] provided conservative predictions with the concrete strength, while, the equation of Rebieiz [21] provided unsafe predictions with the concrete strength. In contrary, the proposed equations provided consistent predictions with the concrete strength.

![Figure 5. Effect of shear span-effective depth ratio on the shear predictions of the proposed and existing equations](image)

8.2 Effect of Shear Span-Effective Depth Ratio

The shear span to effective depth ratio, a/d, is another governing variable that have significant effect on the strength predictions. Fig 5 indicates that, the ACI318-14 equation [2] was scattered with (a/d) ratio, the equations of Niwa et al. [19] and Tureyen and Frosch [22] were conservative with the (a/d) ratio, while, the equation of Rebieiz [21] was unsafe with the (a/d) ratio. The strength predictions of BS8110 equation [3] and EC2 equation [4] were conservative for beams tested with (a/d) below 5, and the strength predictions were consistent with the strength predictions of beams tested with (a/d) above 5. Unlike the above equations, the predictions of the proposed equations and that of Collins and Kuchma [20] were consistent with the (a/d) ratio.
8.3 Effect of Flexural Reinforcement

The effect of the flexural reinforcement ratio, \( \rho \), on the strength predictions of existing and proposed equations is also examined. As can be seen from Fig 6, the shear strength predictions of the ACI318-14 equation [2] and that of Collins and Kuchma [20] were unsafe for beams having flexural reinforcement ratio smaller than 1%, and conservative for beams having flexural reinforcement ratio larger than 1%. The strength predictions of BS8110 [3] and EC2 [4] were consistent for beams having flexural reinforcement ratio smaller than 3%, and conservative for beams flexural reinforcement ratio larger than 3%. The equations of Niwa et al. [19] and Tureyen and Frosch [22] provided conservative predictions with the flexural reinforcement ratio, while, the equation of Rebieiz [21] provided unsafe predictions with the flexural reinforcement ratio. The proposed equations, though, were consistent with the flexural reinforcement ratio.

8.4 Effect of Beam Size

Last, the effect of the beam’s size on the shear strength predictions is examined. It can be inferred from Fig 7 that the equations of ACI318-14 [2] and Collins and Kuchma
[20] provided scattered strength predictions for beams with an effective depth, \( d \), below 600 mm, and provided unsafe predictions for beams with an effective depth above 600 mm. The equations of BS8110 [3], EC2 [4], Niwa et al. [19], and Tureyen and Frosch [22] provided conservative predictions for beams with an effective depth below 300 mm and provided consistent predictions for beams with an effective depth above 300 mm. The proposed equations, however, were consistent with various effective depths.

From the comparisons made above between the proposed and the existing equations in terms of shear strength predictions and the influencing variables, it was found that the proposed equations provided better predictions than the other existing models. This would be attributed to the well recognition of the governing variables on the shear strength of shallow beams without shear reinforcements.

Figure 7. Effect of the beam’s size on the shear predictions of the proposed and existing equations

9. Conclusions

The following conclusions can be drawn from this study:

1. Two equations were proposed to predict the shear strength of shallow beams without shear reinforcement. The first equation was derived from nonlinear regression analysis and the second equation was derived from neural network analysis. The proposed equations achieved good agreements with the test results of 279 beams
from literature. Both equations consider the effect of concrete compressive strength, the flexural reinforcement, the shear span to the beam’s effective depth, and the size effect.

2. The neural network analysis proved again to be a powerful mathematical tool in solving engineering problems. The equation derived from the neural network analysis (equation 2) provided a better prediction to the test results than that derived from the non-linear analysis (equation 1).

3. The proposed equations showed better agreement with the test results of 279 shallow beams from literature than the existing equations. This would be expected since the existing equations do not account for all the parameters that affect the shear strength.

4. Comparisons between the shear strength calculations of the current codes of practice (ACI318-14 [2], BS8110 [3], and EC2 [4]) and the test results of 279 shallow beams revealed that the ACI318-14 shear equation [2] is the most conservative and scattered, while the BS 8110 [3] shear equation is the most consistent.

5. Comparisons between the shear strength calculations of the existing equations from literature and the test results of 279 shallow beams revealed that the shear equations of Niwa et al. [19], Collins and Kuchma [20], and Tureyen and Frosch [22] were conservative, while that of Rebieiz [21] was unsafe.

**Notation**

\( a \)  Shear span.

\( d \)  Effective depth of a beam.

\( a/d \)  Shear span to effective depth ratio

\( E_c \)  Modulus of elasticity for concrete.

\( E_{st} \)  Modulus of elasticity for steel.

\( f_c \)  Cylinder concrete compressive strength.

\( f_{cu} \)  Cube concrete compressive strength.

\( k \)  Derived from bending theory for a singly reinforced beam.

\( k_d \)  Depth factor in EC2 [4].

\( \lambda \)  Modification factor accounts for types of concrete in ACI318-14 [2].

\( n \)  Ratio of steel modulus of elasticity to concrete modulus of elasticity.

\( \rho \)  Flexural reinforcement ratio.

\( v_{test} \)  Test shear strength.

\( v_{pred} \)  Predicated shear strength.
10. References

2. ACI Committee 318. (2014). “Building Code Requirements for Reinforced Concrete (ACI 318-14) and Commentary (ACI 318M-14)”. American Concrete Institute, Farmington Hills, MI, USA, pp. 519.